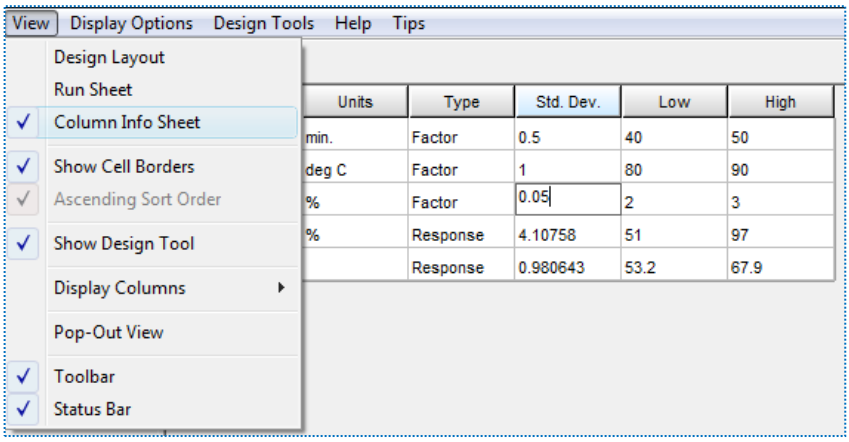
Multifactor RSM Tutorial
(Part 3 – Advanced Topics)

Adding Propagation of Error (POE) to the Optimization

Details about the variation in your input factors can be fed into Design-Expert® software. Then you can generate propagation of error (POE) plots showing how that error is transmitted to the response. Look for conditions that minimize transmitted variation, thus creating a process that's robust to factor settings. This tutorial shows how to generate POE from an experiment designed by response surface methods (RSM).

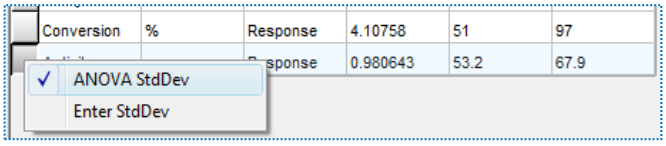
If you still have the RSM data active in Design-Expert software from Part 2 in this series of tutorials, continue on. If you exited the program, re-start it and use **File**, **Open Design** to open your data file (**tut-RSM-opt.dxp**). Otherwise, go back and set it up as instructed in Multifactor RSM Tutorials Parts 1 and 2.

Start by clicking the **Design** node on the left side of the screen to get back to the design layout. Then select **View**, **Column Info Sheet**. Enter the following information into the Std. Dev. column: time: **0.5**, temperature: **1.0**, catalyst: **0.05**, as shown below.



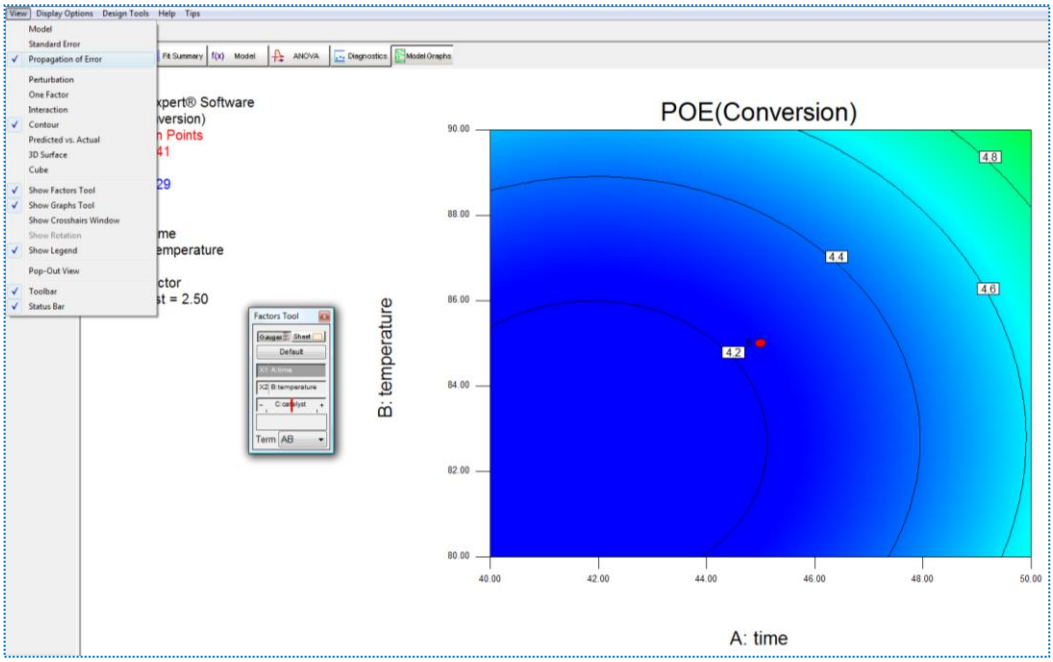
Column Info Sheet with factor standard deviations filled in

Notice the software already entered the standard deviation for the analyzed response, Conversion (4.1...). Because you haven't changed any other data, Design-Expert remembers your previous analysis choices – you can simply click through the analysis buttons. For your information, right-clicking the buttons to the left of response names allows you to specify a different standard deviation.



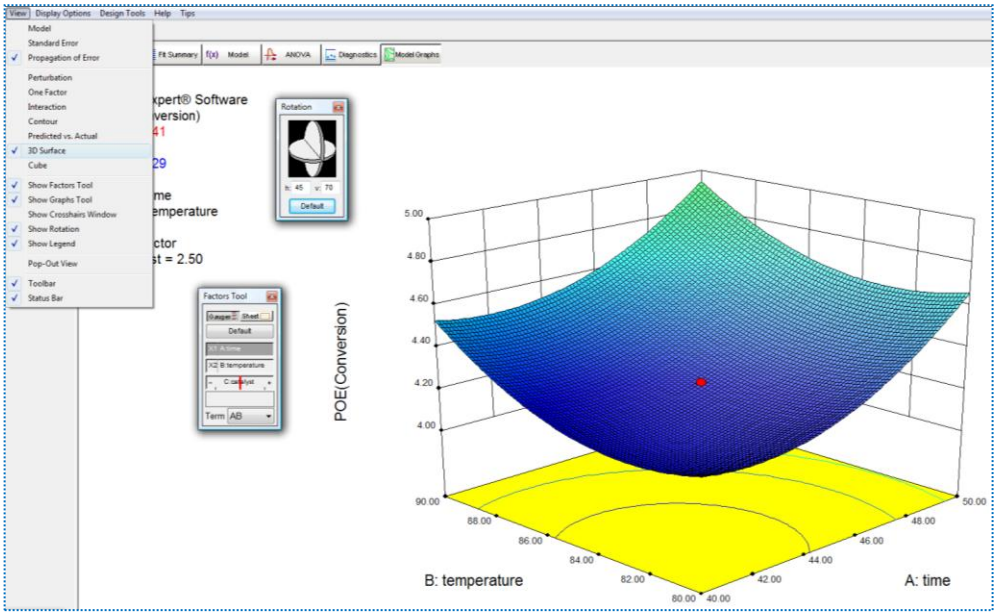
Option to enter a different standard deviation for response

Under the **Analysis** branch click the **Conversion** node. Then jump past the intermediate buttons for analysis and click the **Model Graphs** button. Select **View**, **Propagation of Error**. (This option was previously grayed out – unavailable – because the standard deviations for the factors had not yet been entered.) If the graph's background is solid yellow, switch to Graduated in the Contour graph shading option that's found by right-clicking over the graph and choosing Surface Graphs in Graph Preferences.



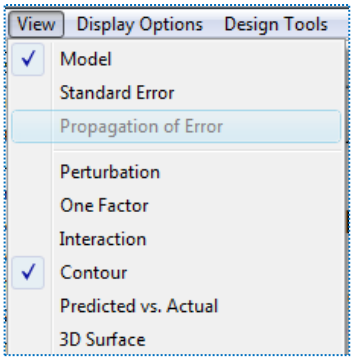
Contour graph for POE

Now select **View**, **3D Surface** or simply click 3D Surface on the floating Graphs Tool.



3D Surface view of the POE Graph

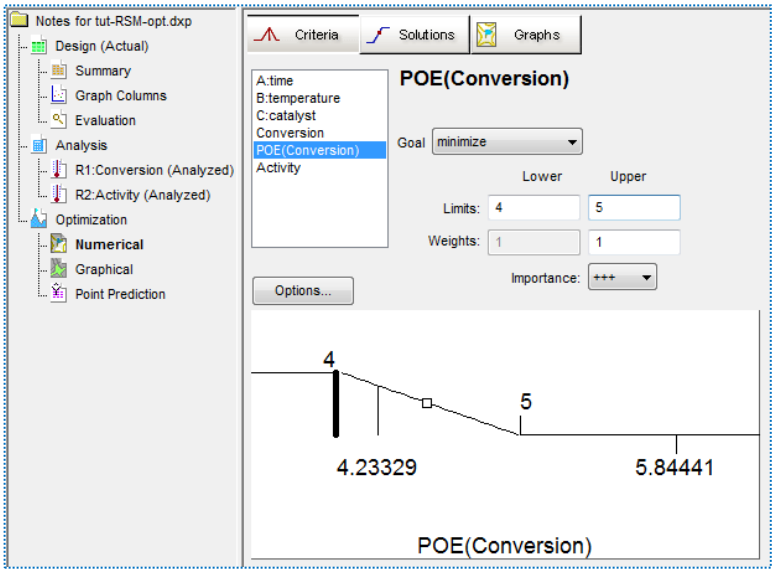
The lower the POE the better, because less of the error in control factors will be transmitted to the selected response, which results in a more robust process. However, POE will only work when the response surface is non-linear, such as for the Conversion. When the surface is linear, such as that for Activity, error is transmitted equally throughout the region. Therefore, Design-Expert software 'grays out' the option as shown in the screen shot below.



POE option not enabled for linear models such as that for Activity response

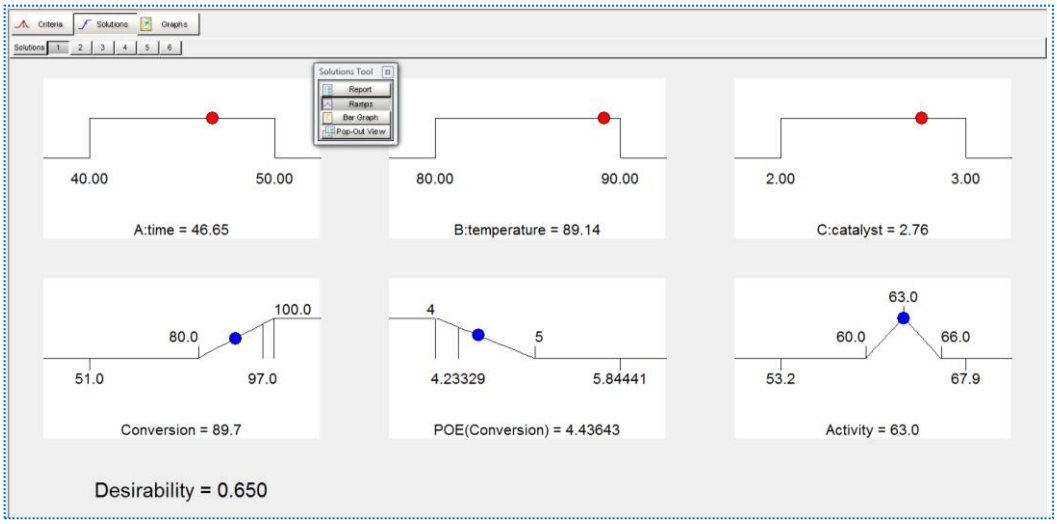
For an in-depth explanation of POE, attend Stat-Ease's workshop Robust Design and Tolerance Analysis. Call, or check our web site for a schedule.

Now that you've generated POE for Conversion, let's go back and add it to the optimization criteria. Under the **Optimization** branch click the **Numerical** node. For the **POE (Conversion)** set the **Goal** to **minimize** with a **Lower Limit** of **4** and an **Upper Limit** of **5** as shown below.



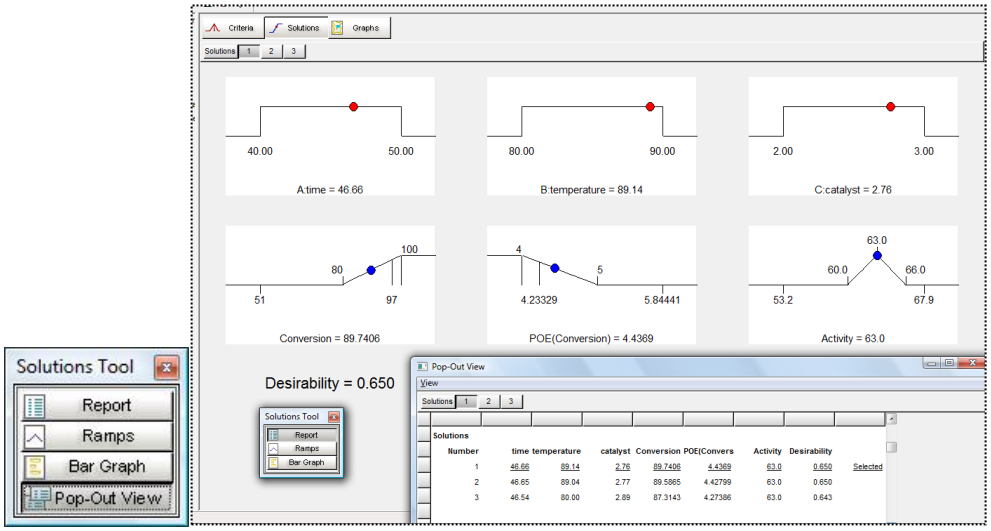
Set Goal and Limits for POE (Conversion)

Now click the **Solutions** buttom atop the screen to generate new solutions with additional criteria. On the floating **Solutions Tool** click **Ramps** or select the View option Ramp Function Graph. (Note: Due to random starting points for the searches, you may see slight differences on your screen versus the shot below.) Click the alternative solutions (2, 3,…). Watch the red dots. What changes do you see?



Ramps view for optimization with POE (Your results may differ)

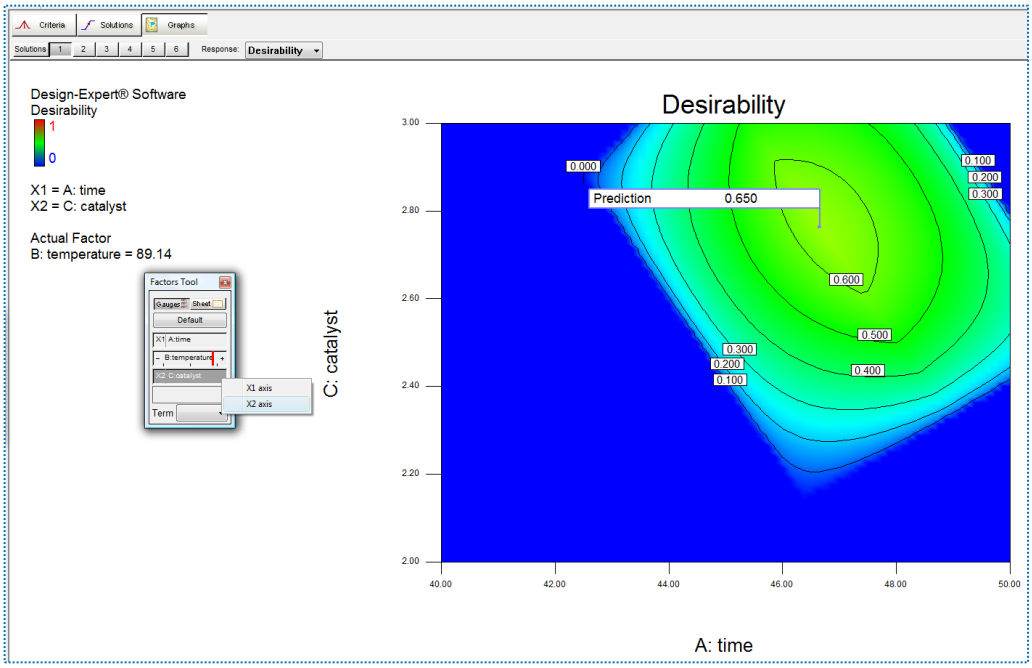
Go back to the **Solutions Tool** and press the **Pop-Out View**.



Pop out view shows report alongside of ramps

Exit out of the Pop-Out View by pressing the **X** button at its upper right corner.

Press the ***Graphs*** button atop your screen. To make the view similar to what we had before, on the **Factors Tool** palette right-click the **C:catalyst** and change it to the **X2 axis**.



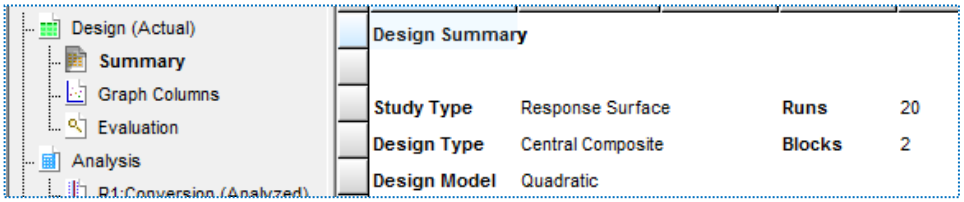
Optimal Solution with added POE criteria

The above optimal solution represents the formulation that best maximizes conversion and achieves a target value of 63 for activity, while at the same time finds the spot with the minimum error transmitted to the responses. So, this should represent process conditions that are robust to slight variations in factor settings.

Design Evaluation

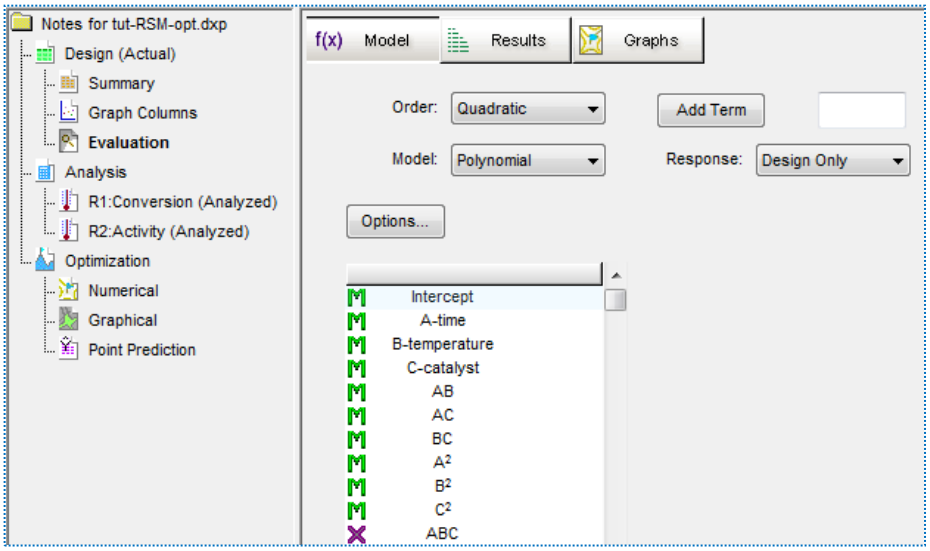
Design-Expert offers powerful tools to evaluate RSM designs. Design evaluation ought to be accomplished prior to collecting response data, but it can be done after the fact. For example, you may find it necessary to change some factor levels to reflect significant deviations from the planned set point. Or you may miss runs entirely – at least for some responses. Then it would be well worthwhile to re-evaluate your design to see the damage.

For a re-cap of what's been done so far, go to the **Design** branch and click the **Summary** node.



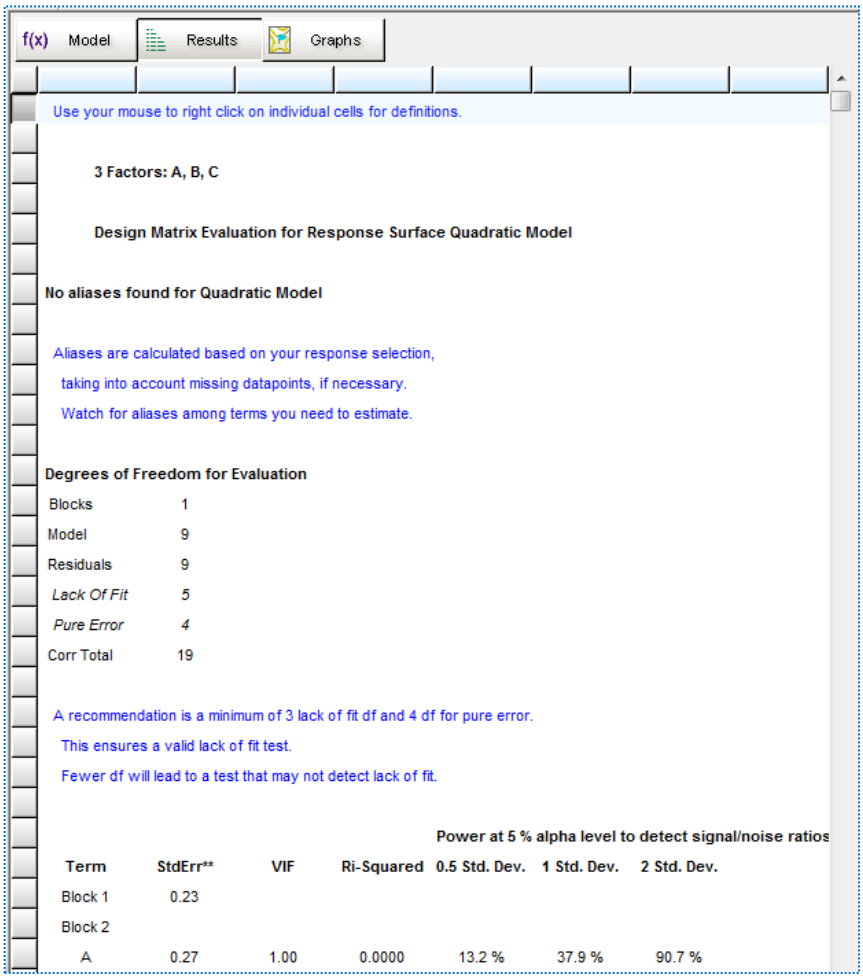
Design summary

The summary reports that the experimenter planned a central composite design (CCD) in two blocks, which was geared to fit a quadratic model. Click the **Evaluation** node and notice Design-Expert assumes you want details on this designed-for order of model.



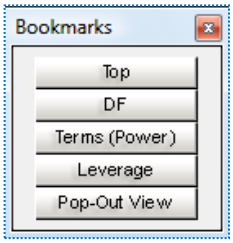
Design evaluation – model choice

Click the ***Results*** button for an initial report showing annotations on by default.



Design evaluation results

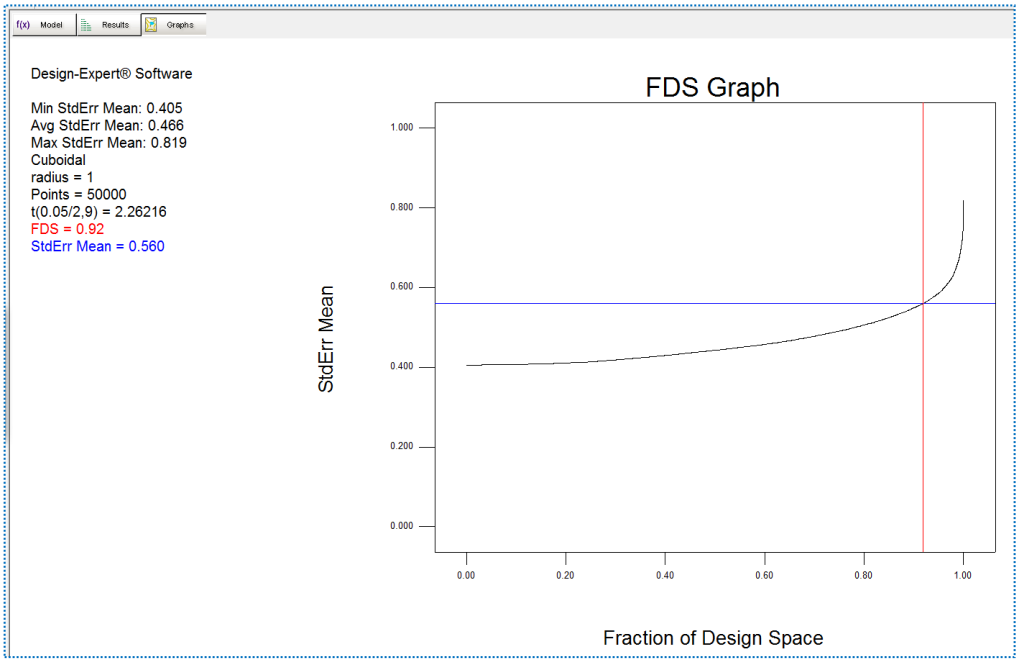
Scroll through the results or use the handy Bookmarks palette and note the results look very good – as you'd expect from a standard design for RSM.



Bookmarks for design evaluation

For a design that produces a far worse evaluation, take a look at the Historical Data RSM Tutorial – Part 2.

Press ahead to the ***Graphs*** button atop the screen. It defaults to the FDS Graph that depicts standard error versus the fraction of design space. Click the curve you see depicted. Design-Expert now provides coordinate lines for easy reading.

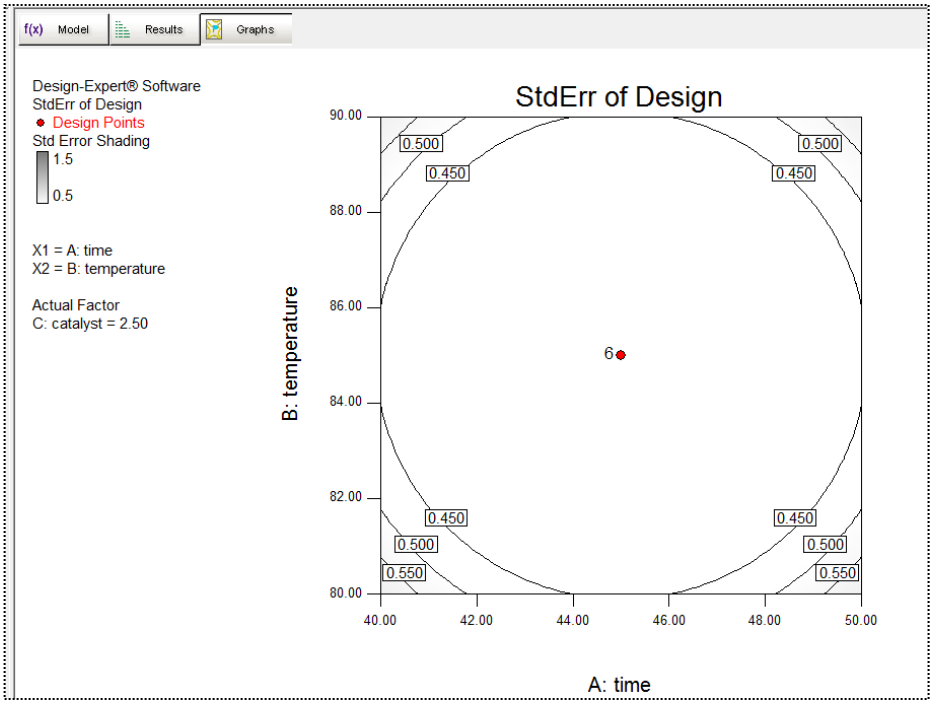


FDS (fraction of design space) graph with coordinates clicked on

Based on extensive sampling of the experimental region (50,000 points by default as noted in the legend), the "y" axis on the FDS graph quantifies the maximum prediction variability at any given fraction of the total space. For example, as noted in the legend at the left of the screen, 90 percent of this response surface method (RSM) design falls at or below 0.560 units of standard error. When you evaluate alternative designs, favor those with lower and flatter FDS curves. However, keep in mind that this provides only one of many criteria that ought to be considered.

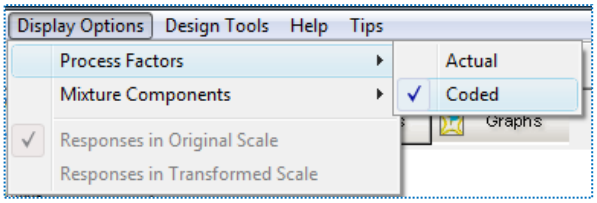
The FDS provides insights on prediction capabilities. To view design 'rotatability' criteria, select **View**, **Contour**. Design-Expert then displays the standard error plot, which shows how variance associated with prediction changes over your design space. You can see the central composite design (CCD) provides relatively

precise predictions over a broad area around the 6 center points. Also, notice the circular contours. This indicates the desirable property of rotatability – equally precise predictive power at equal distances from the center point of this RSM design. (Note: For standard error plots, Design-Expert defaults to black and white shading. The graduated shading that makes normal response contour plots so colorful will not work when displaying standard error.)



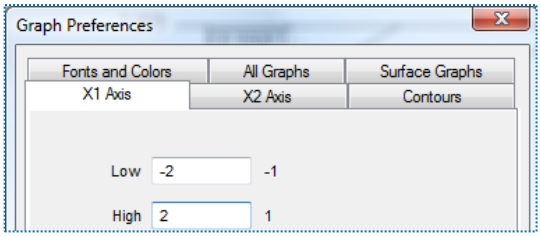
Standard error contour plot

You can manipulate standard error plots just like response plots. Let's see what happens when you extrapolate beyond experimentation regions. Select **Display Options**, **Process Factors**, **Coded**.



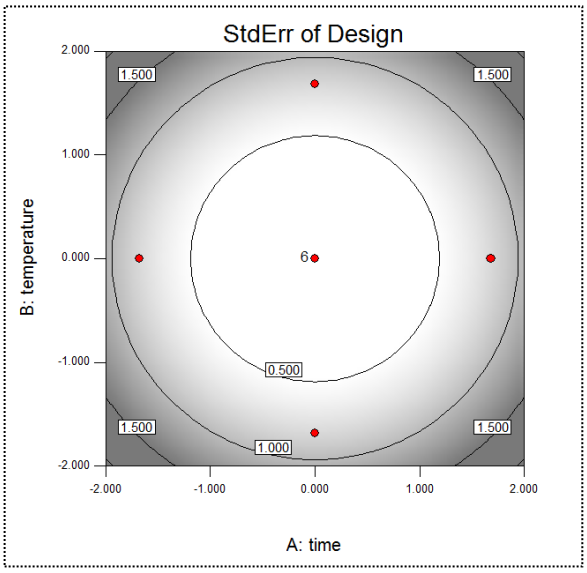
Changing the coding

Now right-click over the graph and select **Graph Preferences**. Change the default **X1 Axis** values for **Low** to **-2** and **High** to **2**.



Changing X1 axis values

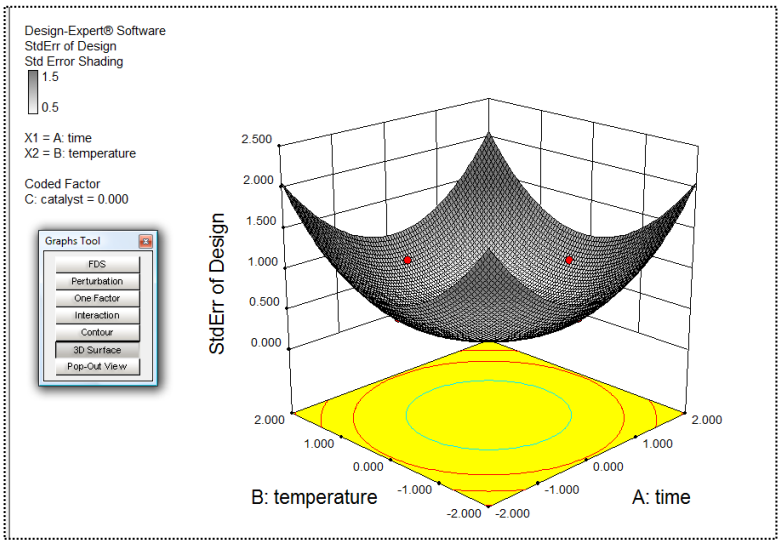
Next, click the **X2 Axis** tab and change **Low** value to **-2** and **High** value to **2**. After completing these changes, press **OK**.



Contour plot of standard error with expanded axes, extrapolated area shaded

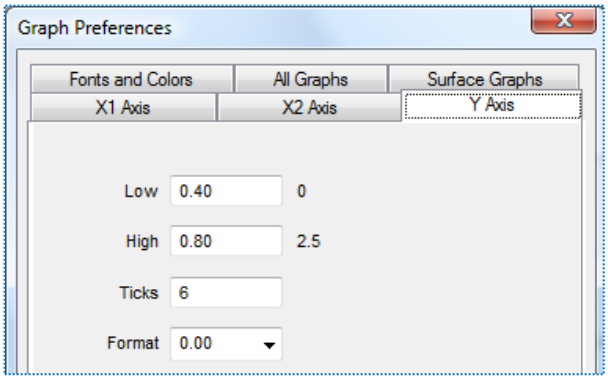
Notice the contour plot corners are shaded. These areas cannot be predicted as well as the interior region. Design-Expert provides shading as a warning against extrapolation. As shown in the key, shading begins at one-half standard deviation and increases linearly up to 1.5 times standard deviation. You also see shading on response (model) plots. So long as you stay within specified factorial ranges, shading remains relatively light, but if you increase the ranges, don't be surprised to see considerable darkening. Be wary of predictions in these nether regions! Remember that darker shading represents higher standard error. You now can see it's dangerous to extrapolate outside plus- or minus-one regions – the factorial portion of this central composite design.

Now on the floating **Graphs Tool** click **3D Surface**.



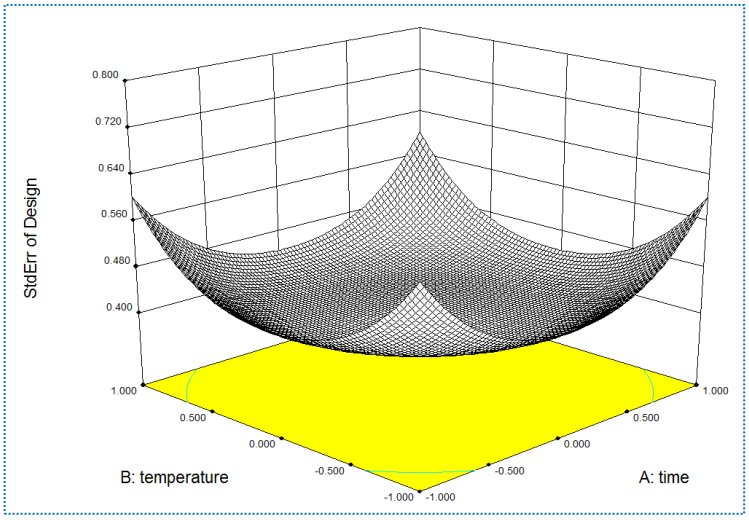
3D view of standard error

This isn't really a good view of the relevant design space because it displays too wide of a range on time and temperature, via a right-click go back to **Graph Preferences** and press the **Defaults** buttons for both **X1 Axis** and **X2 Axis** tabs. This returns the plus- minus-1 coded levels. Also, on the **Y Axis** tab, enter for **Low** the setting of **0.40** and for **High** type in **0.80**.



New settings for Y Axis

Press **OK** to see what happens to the graph.



3D plot of standard error

Notice the flat bottom in this bowl-shaped surface of standard error. That's very desirable for an RSM design. It doesn't get any better than this!