

# Multifactor RSM Tutorial

## (Adding categorical factors)

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### Combining categorical with numeric factors

Procter and Gamble (P&G) engineers working on a new sealing process became concerned about how maximum peel strength would be affected by changing suppliers of packaging material (Brenneman, William A., and William R. Myers, "Robust Parameter Design with Categorical Noise Variables," *Journal of Quality Technology* 35, no. 4 (October 2003): 335-341). They set up an RSM design to vary several key factors on the sealing machine, including the supplier:

- A. Temperature, 193 to 230 degrees C.
- B. Pressure, 2.2 to 3.2 Bar.
- C. Speed, 32 to 50 cpm (cycles per minute).
- D. Supplier: S1, S2, S3.

Due to limitations of time and other resources, a maximum of 37 runs can be performed. Therefore, simply conducting a standard central composite design (CCD) or Box-Behnken design (BBD) for each of the three suppliers will not do – these design choices produce far too many runs (60 and 51, respectively). Instead, P&G engineers use a D-optimal design.

The data below come from a simulation loosely based on the predictive model reported in the cited article. (For tutorial purposes, some liberties are taken to make the procedural outcome more challenging.) Assume the maximum peel strength ideally hits a target of 4.5 pound-force (lbf). However, it must exceed 3 lbf to prevent leaking and not exceed 6 lbf because the package becomes too difficult to open.

This tutorial explains how a design is constructed via D-optimal criteria with added points for lack of fit testing and pure-error estimation. Notice we use nearly all budgeted runs. Also, aided by a feature in the software that forces balance, we make sure each supplier receives an equal number of design runs.

With guidance from the tutorial you analyze responses and then find desirable solutions meeting the following objectives – listed according to relative acceptability for all concerned:

1. Can the process be adjusted to attain a maximum peel strength target of 4.5 lbf for any one supplier?
2. The purchasing agent intends to break down suppliers as follows: 50% to S1, 25% to S2, and 25% to S3. Can you find a process setup that works for all suppliers? If not, a two-supplier option might be satisfactory, provided S1 is one of them. In other words, assuming the process is not robust for all three suppliers, can the process be set up in a manner that either S1-S2 or S1-S3 meets specifications?


3. If it is not possible to achieve a common process for even two of the three suppliers, perhaps the upper limit of 6 can be raised on the maximum peel strength specification. (Assume customers are willing to use scissors or, better yet, package designers notch the plastic for easier opening.) Does this open a window of operability for multiple suppliers at one set of process conditions?

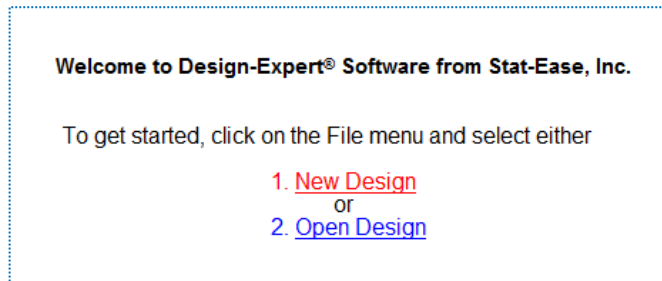
Here is the simulation data loosely based on the above cited article:

<b>Std</b>	<b>A: Temp (deg C)</b>	<b>B: Pressure (Bar)</b>	<b>C: Speed (cpm)</b>	<b>D: Supplier</b>	<b>Peel Strength (lbf)</b>
1	193	2.2	32	S1	4.6
2	230	2.2	32	S1	10.0
3	193	3.2	32	S1	7.2
4	193	3.2	32	S1	6.6
5	230	3.2	32	S1	5.5
6	202.25	2.7	41	S1	8.2
7	230	2.7	41	S1	8.5
8	193	2.2	50	S1	6.7
9	230	2.2	50	S1	11.0
10	230	2.2	50	S1	11.0
11	230	3.2	50	S1	7.8
12	230	3.2	50	S1	7.1
13	230	2.2	32	S2	5.1
14	230	2.2	32	S2	6.7
15	211.5	2.7	32	S2	2.9
16	230	3.2	32	S2	1.7
17	211.5	2.2	41	S2	6.3
18	193	2.7	41	S2	2.0
19	220.75	2.95	41	S2	3.7
20	193	2.2	50	S2	2.4
21	230	2.7	50	S2	4.0
22	230	2.7	50	S2	4.1
23	193	3.2	50	S2	4.8
24	193	3.2	50	S2	5.1
25	193	2.2	32	S3	6.7
26	193	2.2	32	S3	6.7
27	230	3.2	32	S3	8.0
28	230	3.2	32	S3	7.1
29	220.75	2.45	36.5	S3	9.2
30	202.25	2.7	36.5	S3	8.1
31	211.5	3.2	41	S3	7.2
32	230	2.2	50	S3	9.5
33	230	2.2	50	S3	9.8
34	211.5	2.7	50	S3	6.2
35	193	3.2	50	S3	5.9
36	230	3.2	50	S3	4.8

*Data based on package sealing experiment*

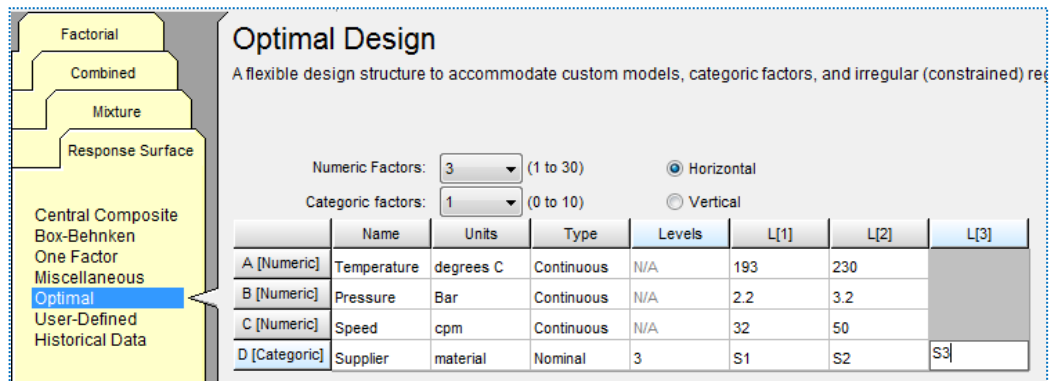
## Design the Experiment

Start Design-Expert® software and initiate your design process by clicking the blank-sheet icon  on the left of the toolbar, or select File, New Design, or simply click **New Design** on our opening screen.



*New easy-start design options*

Click the yellow **Response Surface** folder tab, select **Optimal** as your design. For **Numeric Factors** choose **3** from the droplist. For **Categorical Factors** choose **1**. Key in (and tab over) **Name, Units, L[1]** (lower limit) and **L[2]** (upper limit) for numeric factors A, B, and C as shown below. For categorical factor D, key in (and tab over) **Name, Units, Levels (3), and L[1], L[2], L[3]**.

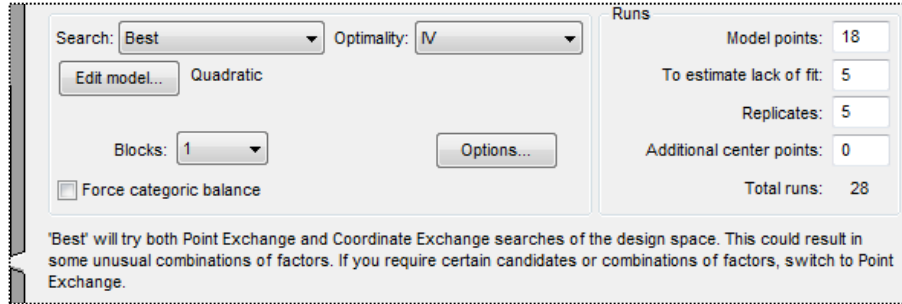


	Name	Units	Type	Levels	L[1]	L[2]	L[3]
A [Numeric]	Temperature	degrees C	Continuous	N/A	193	230	
B [Numeric]	Pressure	Bar	Continuous	N/A	2.2	3.2	
C [Numeric]	Speed	cpm	Continuous	N/A	32	50	
D [Categorical]	Supplier	material	Nominal	3	S1	S2	S3

*Entering numeric and categorical factor levels for optimal design*

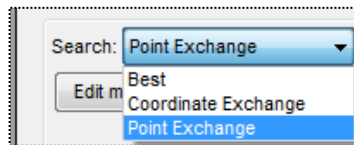
In the above screen shot, the **Type** column automatically selects via default either “Nominal” or “Continuous.” “Nominal” applies to categorical factor D because its treatments are discrete names (S1, S2, S3). When entering ordered treatments (1, 2, 3, for example), it is best to set these up with “Ordinal” contrasts – available as a droplist option – because subsequent model construction and ANOVA layout are then properly applied.

Press **Continue**. Notice the “Search” droplist defaults to “Best” and “Optimality” defaults to “IV.”



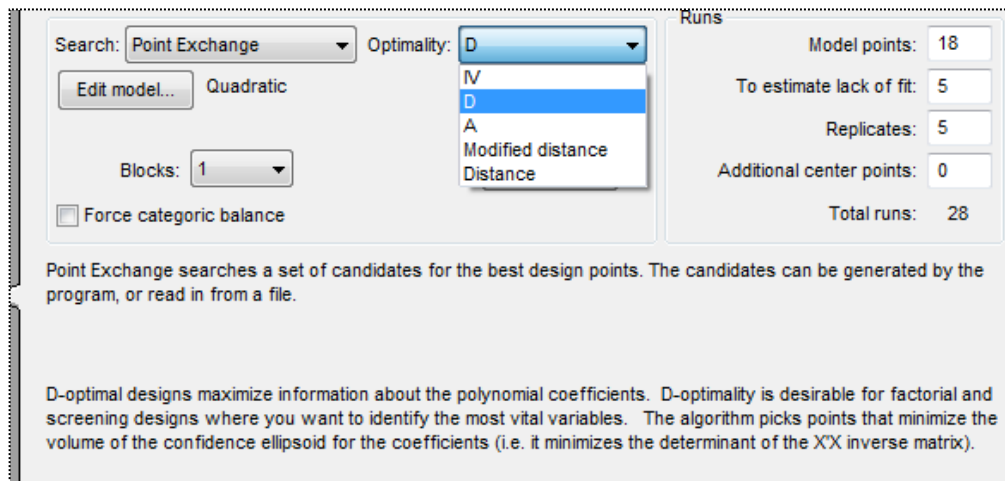
### Default selections for optimal design

To create more convenient levels in the numerical factors, change the **Search** to **Point exchange**.



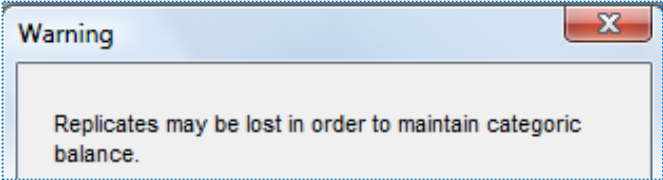
### Changing search to point exchange

Change the **Optimality** to **D**, which may be the better choice for designs with categorical factors as noted in the comments you see on screen. For more statistical details on Design-Expert's optimal model selection screen press the screen tips icon (the light bulb) or select Tips, Screen Tips off the main menu.



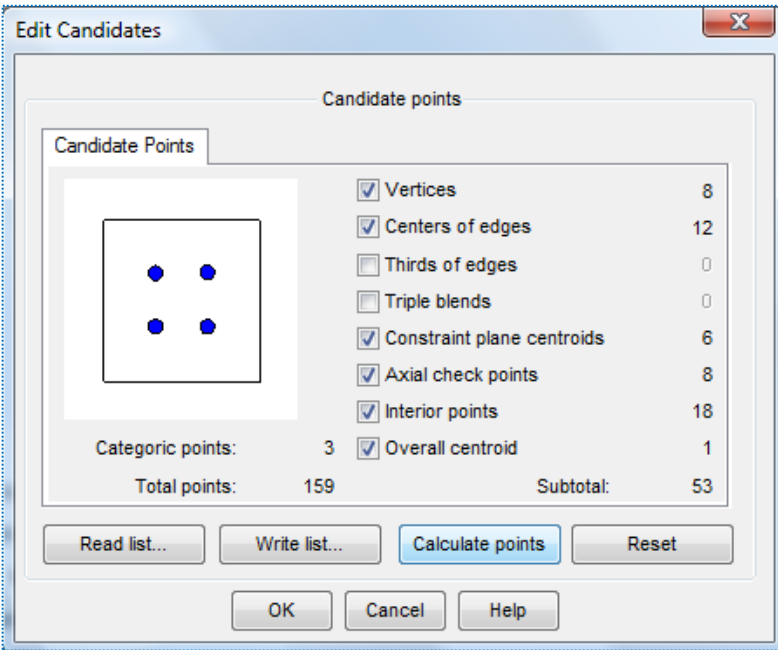
### Changing to D optimality

At this stage the runs total to 28, which is not divisible by 3 – desirable for splitting evenly among your three suppliers. To compensate for this, bulk up your design to 36 runs (12 to each supplier) by increasing **To estimate lack of fit** to **9**. Tab to **Replicates** and again enter **9**. Set a check-mark at **Force categoric balance**. Not doing this could result in a design with more of one supplier than another, which would doubtless generate questions from the purchasing department because it would seem unfair. A warning alerts you that forcing categoric balance might require adjustments.



*Forcing categoric balance might require adjustments*

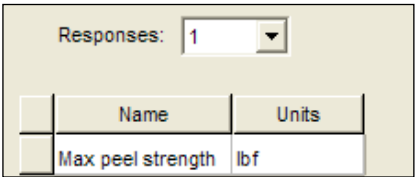
Press **OK** to continue. Now press **Edit candidate points** so you can see which combinations Design-Expert starts with, from which only an optimal subset will be chosen. A pop-up screen appears. Edit this by un-checking “Thirds of edges” and “Triple blends,” which the packaging engineers did not like due to their inconvenient combinations. Then press **Calculate points** to see if this reduced point selection still produces more than the minimum needed to fit the selected model.



*Optimal specifications modified*

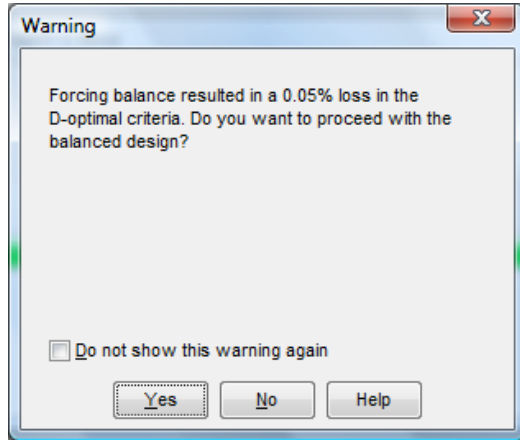
The program now identifies 53 candidate points for each of the 3 suppliers for a total of 159, from which it will select optimally 18 points needed at a minimum for the quadratic model. Design-Expert will add 9 more runs with unique factor combinations to test lack of fit. Finally, 9 of the 27 points already identified will be replicated.

Press **OK**, then press **Continue**. Enter response **Name** and **Units** as shown below.



*Response names and units – name field expanded horizontally*

Press **Continue** again. A warning alerts you Design-Expert is preserving a balanced design for you, which causes a slight loss in optimality.



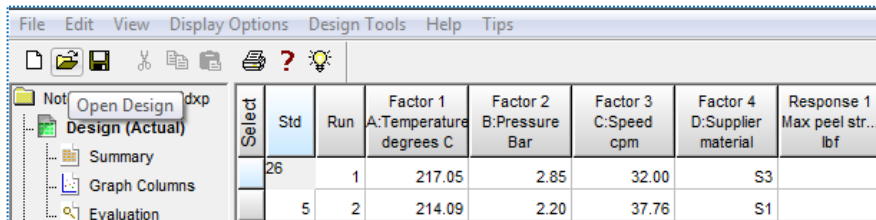
*Forced-balancing warning about a slight loss in optimality*

Your loss percentage may differ from that shown below, but in any case it will be negligible. Choose **Yes** to bring up the design template.

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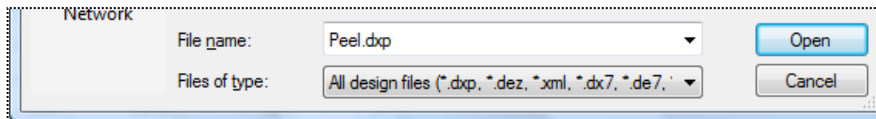
## Analyze the Response

Your layout may match what's shown in the original data table, but there are actually many possible and equally good point selections. To avoid the possibility of a mismatch, but more importantly, to save you time in entering responses, click the open design icon (the one that looks like a folder that's shown below) or select **File, Open Design**.



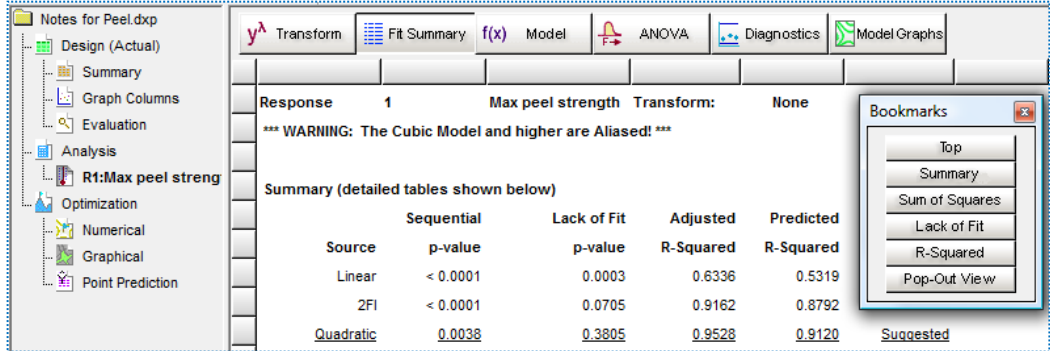
*Opening a design*

Find the file named **Peel.dxp** and open it.



*Opening the file previously created for this case*

Under the **Analysis** branch, click **Max peel strength**. Then press **Fit Summary** atop the screen.



*Fit summary report*

Notice that design-for model, quadratic, is suggested. On the **Bookmarks** palette, click **Lack of Fit**. This looks good (insignificant at  $p > 0.1$ ), so press ahead to **ANOVA** with this chosen model.

ANOVA for Response Surface Quadratic Model  
 Analysis of variance table [Classical sum of squares - Type II]

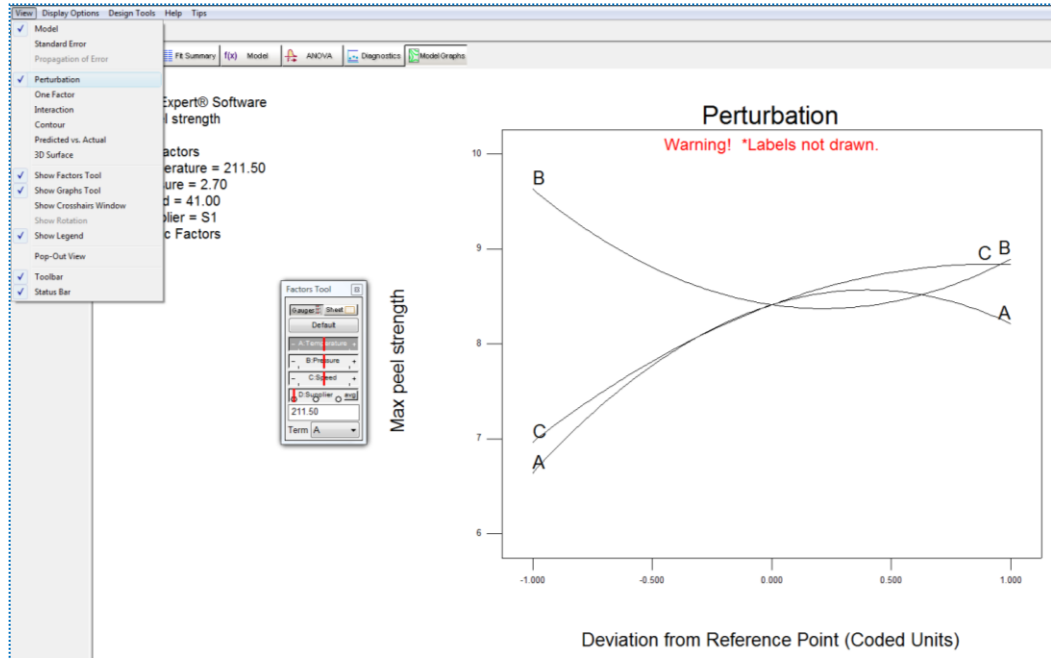
Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F
Model	197.89	17	11.64	42.57	< 0.0001
A-Temperature	17.15	1	17.15	62.73	< 0.0001
B-Pressure	19.69	1	19.69	72.01	< 0.0001
C-Speed	0.017	1	0.017	0.062	0.8058
D-Supplier	89.30	2	44.65	163.30	< 0.0001
AB	44.50	1	44.50	162.76	< 0.0001
AC	0.61	1	0.61	2.22	0.1534
AD	0.63	2	0.31	1.14	0.3409
BC	0.22	1	0.22	0.82	0.3777
BD	1.26	2	0.63	2.31	0.1276
CD	19.28	2	9.64	35.25	< 0.0001
A <sup>2</sup>	2.25	1	2.25	8.24	0.0102
B <sup>2</sup>	2.60	1	2.60	9.50	0.0064
C <sup>2</sup>	0.73	1	0.73	2.68	0.1190
Residual	4.92	18	0.27		
Lack of Fit	2.72	9	0.30	1.23	0.3805
Pure Error	2.21	9	0.25		
Cor Total	202.81	35			

Std. Dev. 0.52 R-Squared 0.9757  
 Mean 6.45 Adj R-Squared 0.9528  
 C.V. % 8.11 Pred R-Squared 0.9120  
 PRESS 17.85 Adeq Precision 25.851

*ANOVA for quadratic model*

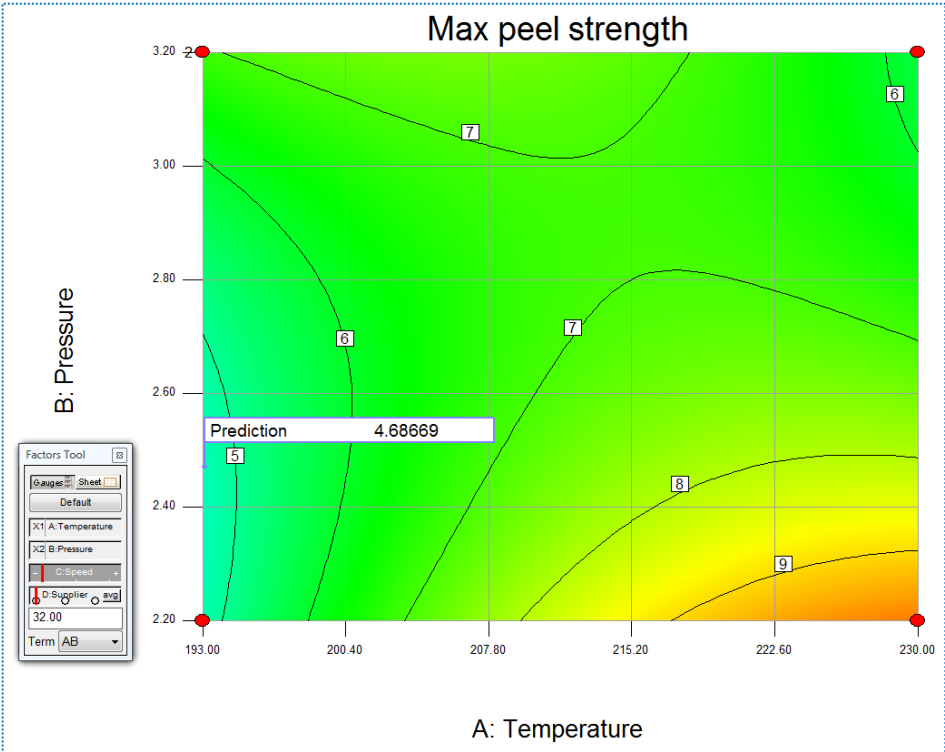
The statistics are excellent. Some terms come out insignificant, but carry them along by pressing **Diagnostics** (look these over – they shouldn’t alarm you). Now view **Model Graphs**. This is where it gets intriguing because the suppliers and purchasing agent are anxious to see the findings! Select **View, Perturbation** (or

click this option on the floating Graphs Tool) to see an overview plot before generating contours and 3D surfaces.



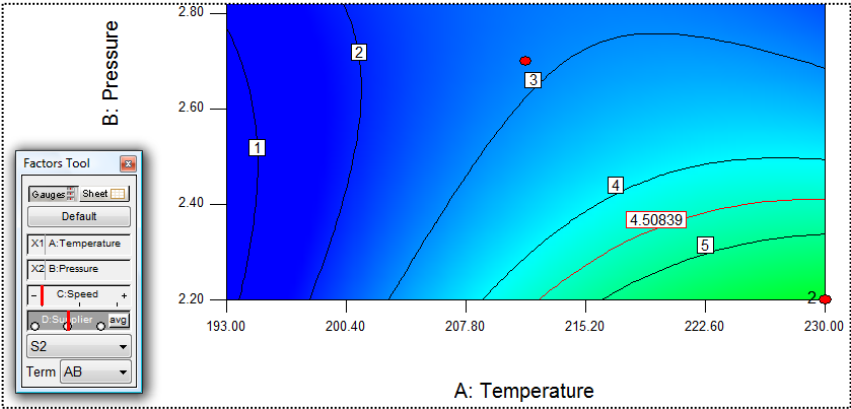
*Perturbation plot*

Remember that 4.5 lbf is the target, so supplier S1 – chosen by default – is coming out high. Lowering factor C will help bring the response down towards the target value. Return to **View, Contour** (or click this plot option on the Graphs Tool). On the **Factors Tool**, click C:Speed and slide it left. At the lowest response levels – the defaulting blue-green graduated color shading – see what’s predicted via a right-click and **Add flag**.



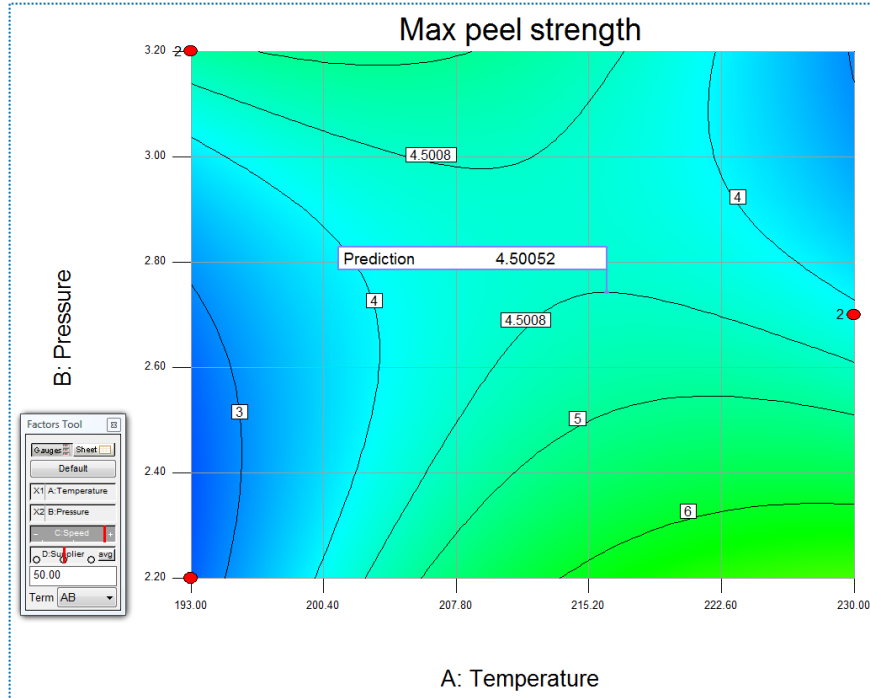
Supplier 1 is not able to produce the target response, even at lowest speed (factor C)

On the **Factors Tool** at **D:Supplier**, click the middle dot to see S2 results. Then right-click the graph, **Add Contour**, and drag it near the required **4.5**.



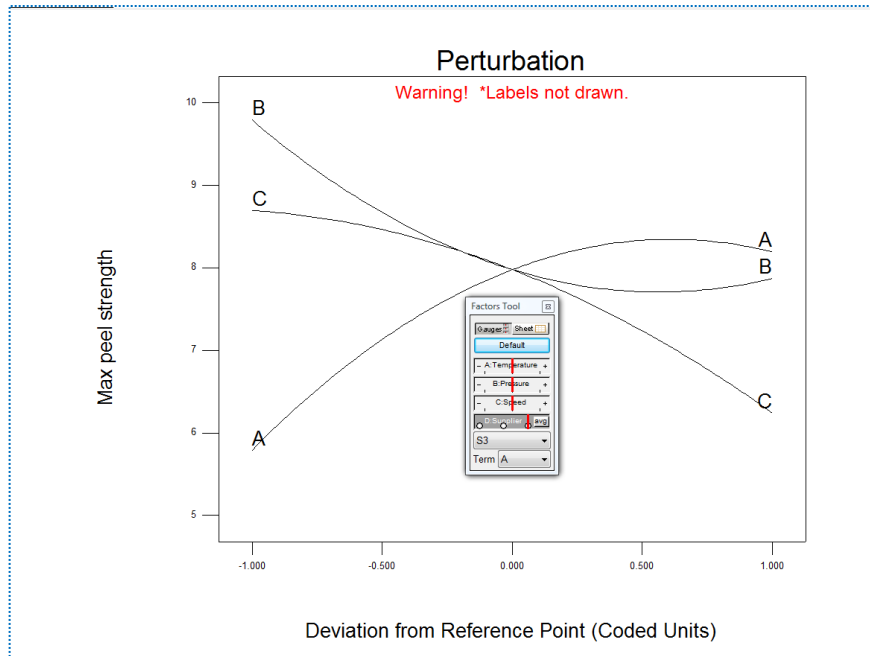
Supplier 2 contour plot

On the **Factors Tool**, click C:Speed and slide it right. Then right-click and **Add flag** near the middle that achieves the response specification. This second supplier (S2) looks much better. With their material, the process need not be pushed too hard in one factor direction or another to meet the critical specification of maximum peel strength on their packaging material.



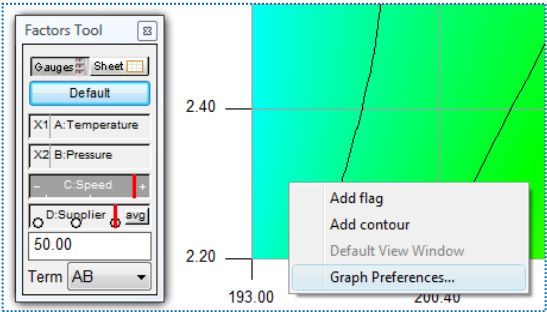
*Supplier 2 is looking good!*

On the **Factors Tool** at **D:Supplier**, click the right dot to see S3 results – it doesn't look good – the peel strength goes to high again. ☹ Select from the Graphs Tool the **Perturbation** plot to get an overview of what's occurring with this supplier. Click Default on the **Factors Tool** to center **C:Speed**. Then click the right dot at **D:Supplier** to reset it for S3.



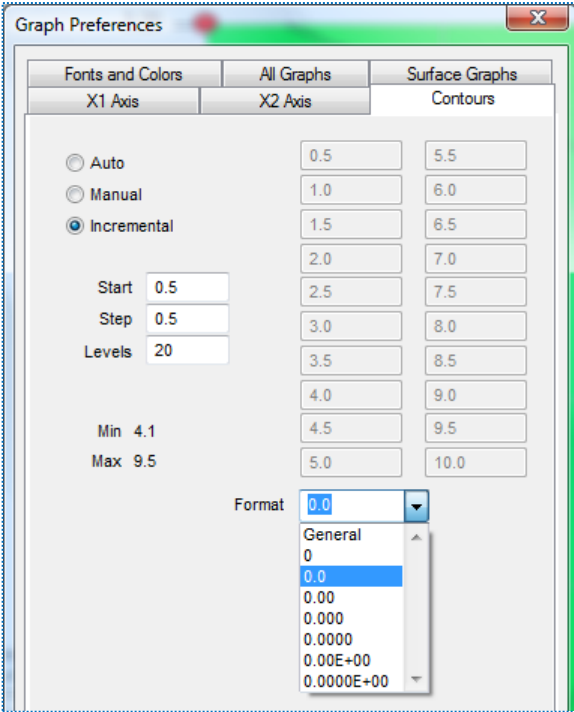
*Perturbation plot for supplier S3*

Notice the high factor C level might allow the 4.5 target to be achieved at certain A and B settings. Go back to **View, Contour**, and on the **Factors Tool** click C:Speed and slide it right. Then right-click the graph and select **Graph preferences**.



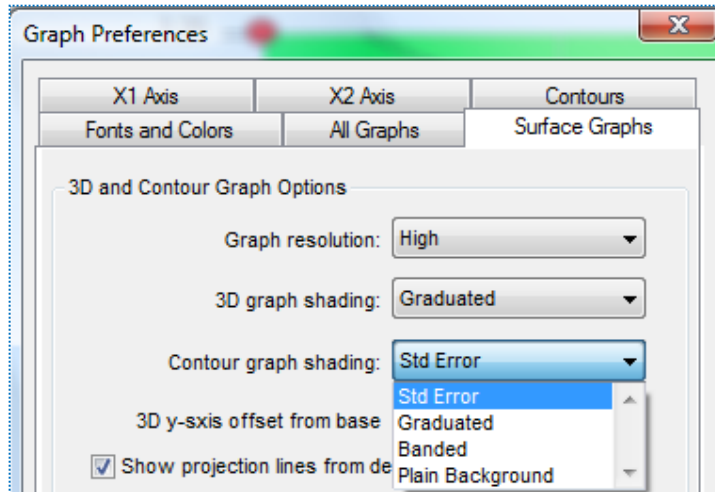
*Third supplier chosen for contour plot with Graph preferences selected*

On the **Contours** tab for graph preferences, select **Incremental** option. Enter a **Start** of **0.5**, **Step** of **0.5**, and set **Levels** at **20** (the maximum). Then change **Format** to **0.0**.



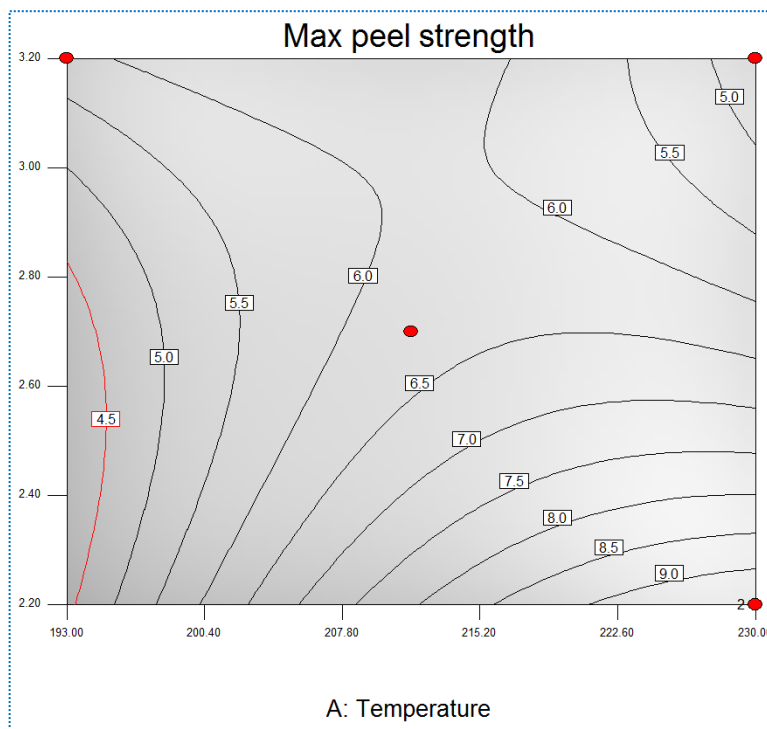
*Changing contour levels*

On the **Surface Graphs** tab, select for **Contour graph shading** the **Std Error** shading option (better for black and white printing).



*Standard error shading selected for contour graph*

Press **OK**. Click the 4.5 contour line and note where the specification is met by this supplier with the proper process setup. (Image below shows contour grid lines turned off via Graph Preferences.)



*The target of 4.5 highlighted as a contour for supplier 3*

The red dots represent conditions where actual experimental runs were performed and, in one case, replicated (as indicated by the “2”). Unfortunately, no points are seen near the desired contour. Thus this region is shaded darker by Design-Expert to indicate a higher standard error for predictions.

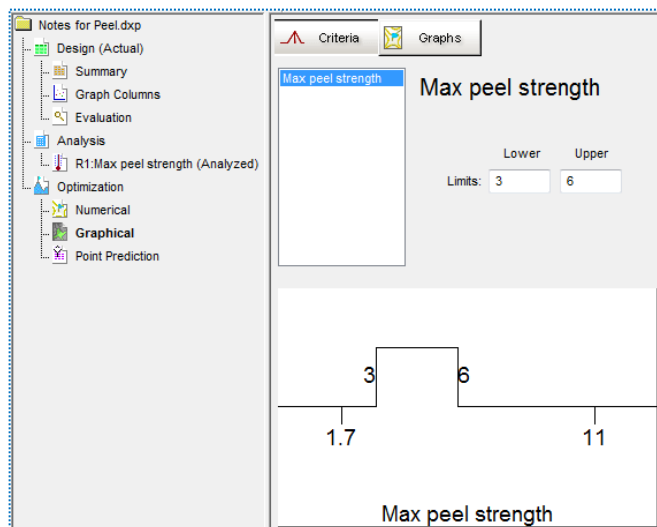
## Graphical Optimization

Recall the questions asked at the outset of this tutorial about the new packaging machine developed by the engineers at P&G. We now know the answer to the first one – yes, the process can be adjusted to hit the maximum peel strength target of 4.5 lbf for one or more suppliers.

The purchasing agent intended to break down the supply in this manner: 50% to S1, 25% to S2, and 25% to S3. This led to the second question: Can you find a process setup that works for all suppliers? The contour plots for S1 make it clear that this supplier, despite being favored by the purchasing agent, will not achieve targeted peel strength within current factor levels. That foils the next most acceptable alternative of a two-supplier option with S1 being one of them. (For some reason, hopefully based on cost, the purchasing agent really likes the first supplier!)

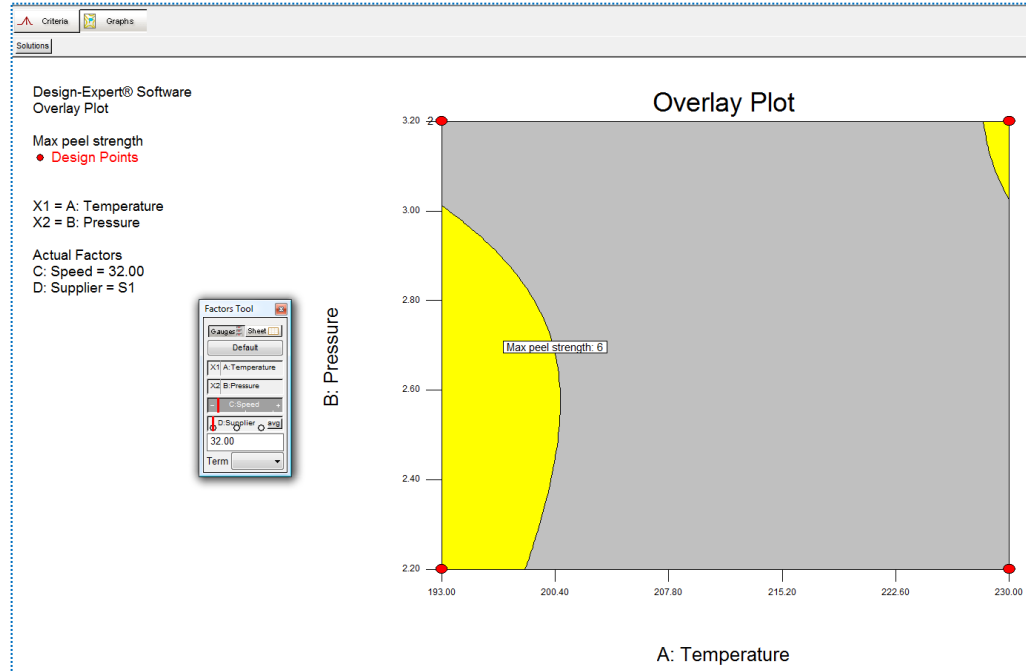
All that's left for the engineers is the following fallback position: If it is not possible to achieve a common setup for even two of the three suppliers, perhaps the upper limit of 6 could be raised on the maximum peel strength specification. (Assume your customers are willing to use scissors or, better yet, package designers can notch the plastic for easier opening.) Will this open a window of operability for multiple suppliers using one set of process conditions?

It's time to explore the operating windows of each of the three suppliers for making packaging that exceeds 3 lbf (to prevent leaking) and not exceed 6 lbf (making the package too difficult to open). Under the **Optimization** branch, click the **Graphical** node. Enter the **Limits at Lower 3 and Upper 6**.



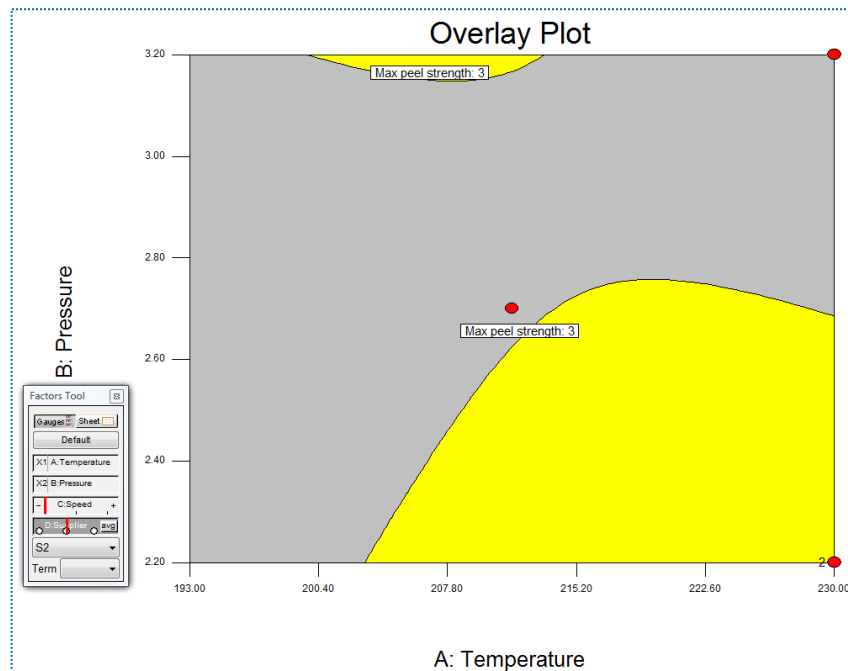
### Criteria for graphical optimization

Click the **Graphs** button. You'll see there's no operating window at the default settings. On **Factors Tool** you must therefore click the bar for **C:Speed** and slide it left, thus opening a sweet spot for favored supplier (S1).



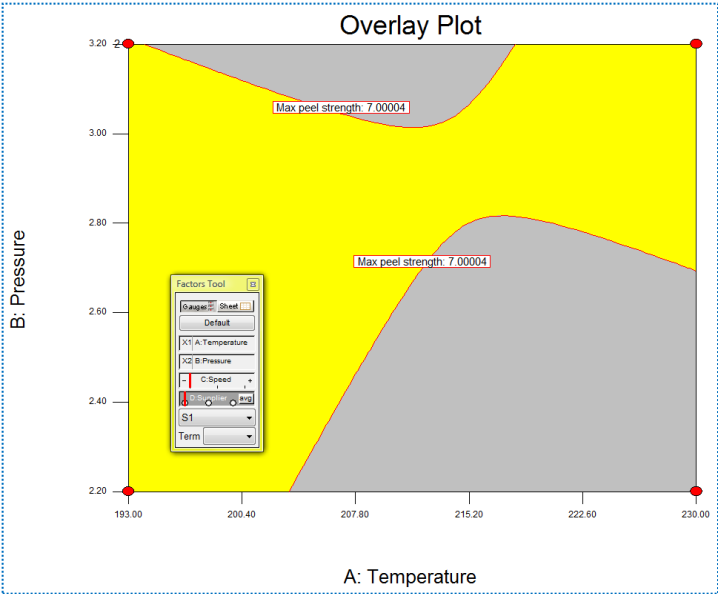
*Sweet spot for supplier 1 at low speed (C)*

Next click D:Supplier dots for alternatives S2 (middle) and S3 (right). Notice only S2 works at this level of C, but it offers no window of opportunity co-existing with S1.



*Window for supplier 2 at same level of factor C*

Now the only remaining alternative is to click the D:Supplier dot for S1 (left dot), then click and drag the border for Max peel strength to the right, causing it to increase above 6. Go ahead and push it all the way up to near 7. (Better have your scissors handy for opening the package!)



*Dragging the border up to 7 for max peel strength*

That's far enough into this case study for tutorial purposes. If you like, click through the other suppliers to see if there's a common condition where even the newly raised packaging specification can be achieved. The answer may surprise you.